Local Class Field Theory

Vatsal Limbachia

September 2020

1 Introduction

Algebraic Number Theory is a study of algebraic number fields, which are finite extensions of $\mathbb{Q}$. We investigate the arithmetic properties of algebraic number fields such as ring of integers, ideals, units, unique factorization etc.

Class Field Theory is the study of abelian extensions of algebraic number fields. These abelian extensions of a field are the Galois extensions of the field with abelian Galois groups.

There are two types of field K that we study in class field theory: local field, $\mathbb{Q}_p$ or $\mathbb{F}_p((t))$ or their finite extensions and global fields, $\mathbb{Q}$ or $\mathbb{F}_p(t)$ or their finite extensions. In this paper, we shall discuss the local field $\mathbb{Q}_p$ and its finite extension K. We shall also refer it to as p-adic local field.

Local Class Field theory is study of abelian extensions of local fields. The local Artin Reciprocity Map is an isomorphism

$$\theta_{/k} : K^{\times} \longrightarrow Gal(K^{sep}/K)^{ab}$$

The study of class field theory was started after the Kronecker-Weber Theorem on abelian extensions of $\mathbb{Q}$. In 1850s, Hilbert proved and built upon the works of Kronecker and Weber. From 1890s to 1920s, there was a lot of development in generalizing number fields. Weber formulated the notions of ray class groups and class fields. Takagi proved that the class fields were the abelian extensions of those given fields. Artin gave a conjecture about reciprocity map and proved it in 1920s and thus established the global class field theory. By 1980s, abelian class field theory had been successfully extended to higher dimensions as well. For non-abelian extensions, it started with ideas of Langland after his letter to Weil in 1967. In this paper, we will only focus on the abelian extensions.

In the modern approach to class field theory, it is stated in terms of ideal class groups and proved using group cohomology. This approach was introduced after the results obtained from the classical approach of Lubin and Tate. In 1930s, Chevalley introduced the notion of adeles and ideles in class field theory. Group cohomology was also being studied in 1930s and 40s. Hochschild and Nakayama reformulated the class field theory in terms of group cohomology and homology in 1950s. Later, Tate introduced the Tate cohomological groups which helped in simplifying the cohomological arguments.

The goal of this paper is to understand the main statements of local class field theory and prove them using cohomology. In the next few sections, we will revise the prerequisites needed and present the statement of local class field theory. In the final section, we prove those statements using techniques of cohomology to give description of Artin Reciprocity Map.

2 Statement of Local Class Field Theory

Before we present the full statement, we will do a quick overview of essential facts from Galois Theory and Local Fields.

Let K be a field

Definition 2.1 A field extension $K \hookrightarrow L$ is called Galois Extension if it is normal, separable, and algebraic.

Definition 2.2 A Galois group $Gal(L/K)$ of a Galois extension is defined as group of automorphisms of L that fix K. This is given by the topology

$$U_S : \{g \in Gal(L/K) : gx = x, x \in S\} : S \in L$$

One can also show that $Gal(L/K)$ is a profinite group as every element $x \in L$ has finitely many Galois conjugates thus making U_x a group with finite index.

Definition 2.3 An absolute Galois group of K is defined as $G_k = Gal(K^{sep}/K)$ where K^{sep} is the separable closure of field K

Theorem 2.4(Fundamental Theorem of Galois Theory) There is an equivalence of categories for he continuous left G_k action to algebras over K which are isomorphic to separable extensions of K, that sends

$$\coprod_i (G_k/H_i) \to \prod_i (K^{sep})^H$$

This functor sends fiber products of G_k sets to tensor product of algebras. It means that for G_k equivariant maps $S_1, S_2 \to T$ of sets, there is an isomorphism

$$F(S_1 \times_T S_2) \cong F(S_1) \otimes_{F(T)} F(S_2)$$

This will aid us to compute tensor products of fields. Thus, it can also be translated that fundamental group Spec$K$ is well defined and isomorphic to the absolute Galois group, as right hand side of tensor product is category of finite etale schemes over Spec$K$.

Definition 2.5 A local field is a field K equipped with an absolute value function $|-|_K : K \to \mathbb{R}$ satisfying the following properties:

1) $|x| = 0$ if and only if $x = 0$

2) there exists an element $x \in K$ such that $x \neq 0, 1$

3) $|xy| = |x||y|$ for all $x, y \in K$

4) $|x + y| \leq |x| + |y|$ for all $x, y \in K$

5) K is complete and locally compact with the topology induced by the metric $d(x, y) = |x - y|$

We define a p-adic local field as a field that is a finite extension of $\mathbb{Q}_p$. It satisfies a stringer triangle inequality; for every $x, y \in K$ we have

$$|x + y| \leq max(|x|, |y|)$$

Proposition 2.6 *If the nonarchimedean and nontrivial absolute value on $|-|$ on K is induced by the discrete valuation v, then the valuation ring A is discrete valuation ring.*

If $\alpha \in K$, then by above proposition, we can write $\alpha = u\pi^r$ with $r \in \mathbb{Z}$, u as a unit and π as the prime element or uniformizer.

If we denote the closed unit ball as

$$\mathcal{O}_K = \{x \in K : |x| \leq 1\}$$

This is know as ring of integers in K. Its maximal ideal is given by open unit ball

$$m_K = \{x \in K : |x| \leq 1\} = (\pi) \subset \mathcal{O}_K$$

The residue field $k = \mathcal{O}_K / m_K$ is a finite field of characteristic p

Proposition 2.7 [CF] *If K is a p-adic local field with absolute value $|-|_K$ and L/K is a finite field extension, then there exists a unique absolute value L that extends $|-|_K$. In particular, it is given by*

$$|x|_K = |N_{L/K}|^{1/d}$$

where $d = [L : K]$ is the degree of field extension

As L/K is a finite extension of p-adic local field, it induces a finite extension of residue fields l/k. We define the ramification index $e_{L/K}$ and inertia degree $f_{L/K}$ as

$$e_{L/K} = [|L^\times| : |K^\times|], f_{L/K} = [l : k]$$

Proposition 2.8 [CF, proposition 5.3] *For any finite extension L/K of p-adic local fields , we have*

$$d = [L : K] = e_{L/k} f_{L/K}$$

Theorem 2.9 [CF, Theorem 7.1] *Let K be a p-adic local field. For every given integer $d \geq 1$, there exists a unique unramified extension L/K of degree f up to isomorphism. In particular, it is given by*

$$L = K(\zeta_{q^f - 1})$$

where $q = |k|$ is the cardinality of the residue field.

As it is a cyclotomic extension, it is Galois and is cyclic. We choose a canonical generator of this group, called Frobenius

$$Frob_{L/K} \in Gal(l/k) \cong Gal(L/K); x \mapsto x^q, x \in L$$

We can also take the union of all the unramified extensions and obtain the maximal unramified extension

$$K^{unr} = \bigcup_{f \geq 1} K(\zeta_{q^f - 1}) \subseteq K^{sep}$$

Theorem 2.10 [CF, Theorem 6.1] Let K be a p-adic local field

1) If L/K is totally ramified extension of degree e, then for any uniformizer $\pi_L = \mathcal{O}_L$, we have $\mathcal{O}_L = \mathcal{O}_K[\pi_L]$. Moreover the monic polynomial of π_L over K is a degree e Eisenstein polynomial with coefficients in $\mathcal{O}_k$

2) If $f(x) \in \mathcal{O}_k$ is an Eisenstein polynomial of degree e, then the splitting field $K[x]/(f(x))$ is a totally ramified extension of degree e. Moreover, all roots in $f(x)$ are uniformizers.

The statement of class field theory is about the abelian part of absolute Galois group. using infinite Galois correspondence, we can see that

$$G^{ab} = Gal(K^{sep}/K)^a b \cong Gal(K^{ab}/K)$$

where the ab denotes the abelianization of the group and K^{ab} is the maximal abelian extension of K

Theorem 2.11 (Local class field theory) For every nonarchimedean local field K, there exists a unique homomorphism:

$$\phi_k : K^{\times} \to Gal(K^{ab}/K)$$

with the following properties:

1) for every prime element π of K and every finite unramified extension L of K, $\phi_K(\pi)$ acts on L as $Frob_{L/K}$

2) for every finite abelian extension L of K, $Nm_{L/K}$ is contained in the kernel $a \mapsto \phi_K(a)|L$, and ϕ_K induces an isomorphism

$$\phi_{L/K} : K^{\times}/Nm_{L/K}(L^{\times}) \to Gal(L/K)$$

In particular,

$$(K^{\times} : Nm_{L/K}(L^{\times})) = [L : K]$$

The map ϕ_K factors as follows

$$\begin{array}{ccc} K^{\times} & \xrightarrow{\phi_k} & Gal(K^{ab}/K) \\ \downarrow & & \downarrow \tau \mapsto \tau|L \\ K^{\times}/Nm(L^{\times}) & \xrightarrow[\phi_{L/K}]{} & Gal(L/K) \end{array}$$

We call ϕ_K and $\phi_{L/K}$ as Local Artin Maps of K and L/K. The subgroups of $K^{\times}$ of the form $Nm(L^{\times})$ for some finite abelian extension L of K are called the norm groups of $L^{\times}$.

3 Cohomology of Groups

In modern number theory, class field theory is proven using the techniques from Galois cohomology. The goal of this section is to gather the tools from cohomology, and use it to produce Artin Reciprocity Map. We will take a similar approach as show in Cassels-Frohlich [CF], Jean-Pierre Serre [Ser] and Milne's notes [Mil].

Let G be a finite group, then it will be a Galois group, with finite extension.

Definition 3.1 A G-module is an abelian group A with a group homomorphism $G \to Aut(A)$. It is also a left $\mathbb{Z}[G]$-module

The group algebra $\mathbb{Z}[G]$ of G is a free abelian group with elements of G as basis and multiplication provided by the group law on G.

Let us denote $G - mod$ as the abelian category of G-modules, and by Ab as abelian category of abelian groups. We observe a inclusion functor $Ab \to G - mod$, where the group gets a trivial G-action on left and right adjoints, called as coinvariants and invariants

$$A \mapsto A_G = A/(a - ga : a \in A, g \in G); A \mapsto A^G = (ga = a, a \in A, g \in G)$$

We can also think of this in following form

$$A_G = \mathbb{Z} \otimes_{\mathbb{Z}} [G]A; A^G = Hom_{\mathbb{Z}[G]}(\mathbb{Z}, A)$$

As $|(-|)_G : G - mod \to Ab$ is a left adjoint, it is right exact. We can take its left derived functors

$$H_i(G, A) = (L_i(-)_G)(A) \in Ab$$

This is called group homology. Similarly, we can define the group cohomology of A as the invariant functor $G - mod \to Ab$ is right adjoints, thus left exact. We consider its right derived functors and get

$$H^i(G, A) = (R^i(-)^G)(A) \in Ab$$

By the property of derived functors, any short exact sequence $0 \to A \to B \to C \to 0$ of G-modules induces a long exact sequence

$$... \to H_1(G, A) \to H_1(G, B) \to H_1(G, C) \to A_G \to B_G \to C_G \to 0$$

$$0 \to A^G \to B^G \to C^G \to H^1(G, A) \to H^1(G, B) \to H^1(G, C) \to ...$$

For a G-module M, we define the norm map $Nm_G : M \to M$ as

$$m \mapsto \textstyle\sum_{g \in G} gm$$

Tate defined

$$H_T^r = \begin{cases} H^r(G, M) & r > 0 \\ M^G/Nm_G(M) & r = 0 \\ Ker(Nm_G)/I_G M & r = -1 \\ H_{-r-1}(G, M) & r < -1 \end{cases} \tag{1}$$

Thus, the exact sequence now forms

$$0 \to H_T^{-1}(G, M) \to H_0(G, M) \to H^0(GM) \to H_T^0(G, M) \to 0$$

The groups $H_T^r(G, M)$ are known as Tate Cohomology groups. For any short exact sequence of G-modules

$$0 \to M^{'} \to M \to M^{"} \to 0$$

After applying extended snake lemma, we get a long exact sequence

$$... \to H_T^r(G, M^{'}) \to H_T^r(G, M) \to H_T^r(G, M^{"}) \xrightarrow{\delta} H_T^{r+1}(G, M) \to ...$$

Proposition 3.2: [Mil] *Let G be a cyclic group of finite order. The choice of generator for G determines the isomorphism*

$$H_T^r(G,M) \xrightarrow{\cong} H_T^{r+2}(G,M)$$

for all G-modules M , $r \in \mathbb{Z}$

Proof: Let σ be the generator of G. Then the following sequence is exact

$$0 \to \mathbb{Z} \xrightarrow{m \mapsto \sum_{g \in G} gm} \mathbb{Z}[G] \xrightarrow{\sigma - 1} \mathbb{Z}[G] \xrightarrow{\sigma^i \mapsto 1} \mathbb{Z} \to 0$$

As the groups in the sequence and the kernel I_G of $\mathbb{Z}[G] \to \mathbb{Z}$, the sequence remains exact even after it is tensored with M. Thus

$$0 \to M \to \mathbb{Z}[G] \otimes_G M \to \mathbb{Z}[G] \otimes_G M \to M \to 0$$

is an exact sequence of G-modules. We know that $\mathbb{Z}[G] \otimes_G M \approx \mathbb{Z}[G] \otimes_G M_0$, where M_0 is the abelian group as M. So, $H_T^r(G, \mathbb{Z} \otimes_G M) = 0$. Thus, we can say that, for all r

$$H_T^r(G,M) \xrightarrow{\cong} H_T^{r+2}(G,M)$$

□

Theorem 3.3 (Tate's Theorem) Let G be a finite group and let C be a G-module. Suppose that for all subgroups H of G (including $H = G$),

a) $H^1(H,C) = 0$

b) $H^2(H,C)$ is a cyclic group of order equal to $(H : 1)$

Then for all r there is an isomorphism

$$H_T^r(G,\mathbb{Z}) \to H_T^{r+2}(G,C)$$

depending only on choice of generator of $H^2(G,C)$

The book [Weiss] gives a complete and detailed account of such theorems of Tate cohomology groups

Remark 3.4[Mil] If M is a G-module, and $Tor_1^{\mathbb{Z}}(M,C) = 0$, which means either M or C is a torsion-free $\mathbb{Z}$-module, then we can tensor the sequence with M and obtain an isomorphism

$$H_T^r(G,M) \to H_T^{r+2}(G,M \otimes C)$$

Example 3.5[Mil] Let K be a local field. We shall prove that for any finite Galois extension L of K with Galois group G, $H^2(G, L^\times)$ is cyclic of order $[L : K]$ with a generator $u_{L/K}$. From Hilbert's Theorem, we know that $H^1(G, L^\times) = 0$. Tate's theorem shows that the cup-product with $u_{L/K}$ is an isomorphism

$$G^{ab} = H_T^{-2}(G, \mathbb{Z}) \to H_T^0(G, L^\times) = K^\times / NmL^\times$$

If we take the inverse isomorpshim, we get the local Artin map. With similar arguments, we can also obtain global Artin map.

Now that we have the necessary tools from cohomological algebra, we will integrate them with local class field theory results and proceed towards proving the local Artin map.

4 Local Class Field Theory via Cohomology

In this section, we will develop the cohomological approach to local class field theory and proceed to prove the existence of local Artin map. Throughout this section, "local field" means "nonarchimedean local field". For a Galois extension of field L/K(could be infinite) set

$$H^2(L/K) = H^2(Gal(L/K), L^\times)$$

Let K be a local field

Proposition 4.1 [Mil] Let L/K be a finite unramified extension with Galois group G and let U_L be group of units in L. Then

$$H_T^r(G, U_L) = 0 \text{ for all } r$$

Proposition 4.2 [Mil] Let L/K be a finite unramified extension. Then the norm map $Nm_{L/K} : U_L \to U_K$ is surjective.

Corollary 4.3 [Mil] Let L/K be an infinite unramified extension with Galois group G. Then $H^r(G, U_L) = 0$ for $r > 0$ (continuous cochains).

Let L be an unramified extension of K and let $G = Gal(L/K)$. As $H^2(G, U_L) = 0 = H^3(G, U_L)$, the cohomology sequence of short exact sequence

$$0 \to U_L \to L^\times \xrightarrow{ord_L} \mathbb{Z} \to 0$$

gives an isomorphism

$$H^2(G, L\times) \xrightarrow[\cong]{H^2(ord_L)} H^2(G, \mathbb{Z})$$

The groups $H^r(G, \mathbb{Q})$ are torsion for $r > 0$ and $\mathbb{Q}$ is divisible, the group is uniquely divisible and hence is 0. Thus, we can produce a short exact sequence

$$0 \to \mathbb{Q} \to \mathbb{Z} \to \mathbb{Q}/\mathbb{Z} \to 0$$

which would yield an isomorphism

$$H^1(G, \mathbb{Q}/\mathbb{Z}) \xrightarrow{\delta} H^2(G, \mathbb{Z})$$

We know that

$$H^1(G, \mathbb{Q}/\mathbb{Z}) \cong Hom_{cts}(G, \mathbb{Z})$$

The Frobenius element $\sigma = Frob_{L/K}$ will act as a generator. Its composite

$$H^2(L/K) \xrightarrow[\cong]{ord_L} H^2(G, \mathbb{Z}) \xleftarrow[\cong]{\delta} H^1(G, \mathbb{Q}/\mathbb{Z}) \cong Hom_{cts}(G, \mathbb{Q}/\mathbb{Z}) \xrightarrow{f \mapsto f(\sigma)} \mathbb{Q}/\mathbb{Z}$$

is called an invariant map

$$inv_{L/K} : H^2(L/K) \to \mathbb{Q}/\mathbb{Z}$$

Theorem 4.4 [Mil] There exists a unique isomorphism

$$inv_K : H^2(K^{un}/K) \to \mathbb{Q}/\mathbb{Z}$$

with a propoerty that for every $L \subset K^{un}$ of finite degree n over K. The inv induces an isomorphism

$$inv_{L/K} : H^2(L/K) = \frac{1}{[L:K]}\mathbb{Z}/\mathbb{Z}$$

Let L be a finite unramified extension of K and Galois group G and let $n = [L : K]$. Let us denote $u_{L/K}$ as the local fundamental class. It is the element of

$H^2(L/K)$ mapped into the generator $1/[L:K]$ of $\frac{1}{[L:K]}\mathbb{Z}/\mathbb{Z}$ from the invariant map. The pair $(G, L^\times)$ satisfies the hypotheses of Tate's theorem and a cup-product with the fundamental class denotes an isomorphism

$$H_T^r(G,\mathbb{Z}) \to H_T^{r+2}(G, L^\times)$$

for all $r \in Z$. For $r = -2$, it becomes

$$\begin{array}{ccc} H^{-2}(G,\mathbb{Z}) & \xrightarrow{\cong} & H^0(G,L^\times) \\ \downarrow & & \downarrow \\ G & & K^\times/NmL^\times \end{array}$$

We now compute this map explicitly.

A prime element π of K is also prime in L and defines a decomposition

$$L^\times = U_L \cdot \pi^{\mathbb{Z}} \cong U_L \times \mathbb{Z}$$

of G-modules. Thus

$$H^r(G, L^\times) \cong H^r(G, U_L) \otimes H^r(G, \pi^Z)$$

We choose the Frobenius generator σ of G and let

$$f \in H^1(G, \mathbb{Q}/\mathbb{Z}) \cong Hom(G, \mathbb{Q}/\mathbb{Z})$$

be the element such that $f^{\sigma^i} = \frac{i}{n}\mathbb{Z}$ for all i. It generates $H^1(G,\mathbb{Q}/\mathbb{Z})$

From the short exact sequence

$$o \to \mathbb{Z} \to \mathbb{Q} \to \mathbb{Q}/\mathbb{Z} \to 0$$

and we know that $H^r(G,\mathbb{Q}) = 0$ for all r, we obtain an isomorphism

$$\delta : H^1(G, \mathbb{Q}/\mathbb{Z}) \to H^2(G, \mathbb{Z})$$

To construct δf, we choose a lifting of f to 1-cochain $\tilde{f} : G \to \mathbb{Q}$. We take $\tilde{f}$ to be the map $\sigma^i \to \frac{i}{n}$, where $0 \leq i \leq n-1$

Then,

$$d\tilde{f}(\sigma^i, \sigma^j) = \sigma^i \tilde{f}(\sigma^j) - \tilde{f}(\sigma^{i+j}) + \tilde{f}(\sigma^i) = \begin{cases} 0 & i+j \leq n-1 \\ 1 & i+J > n-1 \end{cases}$$

We can find the fundamental class $u_{L/K}$ with the help of π^z which is a subgroup of $L^[\times]$. It is represented by the cocycle

$$\varphi(\sigma^i, \sigma^j) = \begin{cases} 0 & i+j \leq n-1 \\ \pi & i+J > n-1 \end{cases}$$

From the short sequence

$$0 \to I \to \mathbb{Z}[G] \to \mathbb{Z} \to 0$$
$$0 \to L^\times \to L^\times(\varphi) \to I \to 0$$

We obtain the following boundary maps

$$H^{-2}(G, \mathbb{Z}) \to H^{-1}(G, I)$$
$$H^{-1}(G, I) \to H^0(G, L^\times)$$

which are isomorphisms due to the trivial cohomology of $\mathbb{Z}[G]$ and $L^\times(\varphi)$ Finally, $H^{-2}(G, \mathbb{Z}) = H_1(G, \mathbb{Z}) \cong G$.

Now with the above results, we have moved very close to proving the local class field theory's main statement. From the Tate's theorem which is satisfied by $(G, L^\times)$, we have proved the following result.

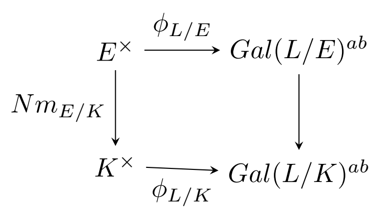
Theorem 4.5 For every finite Galois extension of local fields L/K and $r \in \mathbb{Z}$, the homomorphism

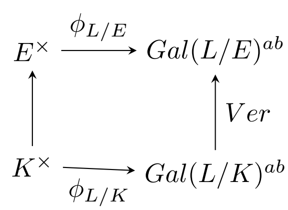
$$H_T^r(Gal(L/K), \mathbb{Z}) \to H_T^{r+2}(Gal(L/K), L^\times)$$

defined by $x \mapsto x \cup u_{L/K}$ is an isomorphism. When $r = -2$, this becaomes an isomorphism

$$G^[ab] \cong K^\times / Nm_{L/K}(L^\times)$$

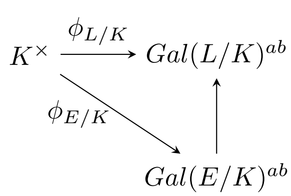
Lemma 4.6 Let $K \subset E \subset L$ be local fields. Then the following diagrams commute





The unmarked arrows are induced by the inclusions $K \subset E$

Let $K \subset E \subset L$ be local fields with both L and E Galois over K. The following diagram commutes



The unmarked arrow is induced by surjection $\sigma \to \sigma|E$

In particular if $K \subset E \subset L$ is a tower of finite abelian extensions of K then $\phi_{L/K}(a)|E = \phi_{E/K}(a)$ for all $a \in K^\times$, thus we can define $K^\times \to Gal(K^{ab}/K)$ to be homomorphism such that, for every finite abelian extension L/K, $\phi_K(a)L = \varphi_{L/K}(a)$

Theorem 4.6 (Local class field theory) For every nonarchimedean local field K, there exists a unique homomorphism:

$$\phi_k : K^\times \to Gal(K^{ab}/K)$$

with the following properties:

1) for every prime element π of K and every finite unramified extension L of K, $\phi_K(\pi)$ acts on L as $Frob_{L/K}$

2) for every finite abelian extension L of K, $Nm_{L/K}$ is contained in the kernel $a \mapsto \phi_K(a)|L$, and ϕ_K induces an isomorphism

$$\phi_{L/K} : K^\times/Nm_{L/K}(L^\times) \to Gal(L/K)$$

Proof: Almost everything is obvious now and follows from the previous results except 1). It follows that L is an unramified extension of K. Recall the result of local fundamental class of Tate's Theorem. Under the following diagram

$$\begin{array}{ccc} H^{-2}(G,\mathbb{Z}) & \xrightarrow{\cong} & H^0(G,L^\times) \\ \downarrow & & \downarrow \\ G & & K^\times/NmL^\times \end{array}$$

The Frobenius element $\sigma \in G$ maps to the class to the class of π in $K^\times/NmL^\times$
□

5 References

[CF] J. W. S. Cassels and A. Fröhlich, Algebraic number theory, Proceedings of an instructional conference organized by the London Mathematical Society (a NATO Advanced Study Institute) with the support of the International Mathematical Union. Edited by J. W. S. Cassels and A. Fröhlich, Academic Press, London; Thompson Book Co., Inc., Washington, D.C., 1967. MR 0215665

[Dwo] Bernard Dwork, Norm residue symbol in local number fields, Abh. Math. Sem. Univ. Hamburg 22 (1958), 180–190. MR 0098079

[Mil] J.S. Milne, Class field theory (v4.02), www.jmilne.org/math/, pp. 281+viii.
[Ser79] 2013,Jean-Pierre Serre, Local fields, Graduate Texts in Mathematics,

vol. 67, Springer-Verlag, New York-Berlin, 1979, Translated from the French by Marvin Jay Greenberg. MR 554237

[Sna] Victor P. Snaith, Galois module structure, Fields Institute Monographs, vol. 2, American Mathematical Society, Providence, RI, 1994. MR 1307386

[Weiss] Weiss , E. 1969. Cohomology of groups. Pure and Applied Mathematics, Vol. 34. Aca- demic Press, New York.